High-frequency electromagnetic field analysis by COCR method using anatomical human body models

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The authors have investigated a high-accuracy analysis for the electromagnetic field of anatomical human body models using the parallel finite element method. In the high-frequency electromagnetic field analysis by the parallel finite element analysis with anatomical human models, efficient conversion from voxels to finite element meshes and speed-up of an analysis become important topics. In this paper, we propose parallel mesh generation method, and show to be able to generate large-scale mesh very effective. Then, we evaluated the performance of two kinds of iterative method by numerical experiments using large-scale human body model. As the results, the COCR method is very useful as the interface solver in the DDM for high-frequency electromagnetic field analysis using anatomical human model.

***Index Terms*—Complex symmetric systems, iterative methods, minimal residual method, parallel computing.**

I. Introduction

In our research group, we are investttigateing and developing full-wave electromagnetic analysis method for large-scale analysis that has several hundred million meshes based on the parallel finite element method [1]. To apply the parallel finite element analysis code in the computation using the anatomical human model, a parallel mesh generation algorithm is verified in the viewpoint of performances using a super computer system.

On the other hand, the high-frequency electromagnetic problem with our formulation is complex symmetric, and we have employed the conjugate orthogonal conjugate gradient (COCG) method for the interface problem. However, because of the ill-conditioned coefficient matrix of the large-scale analysis, it suffers from low convergence rate. To solve this issue, we propose a domain decomposition method (DDM) algorithm based on the conjugate orthogonal conjugate residual (COCR) method [2]. As shown in this paper, an improvement of the performance by using COCR method with huge-scale anatomical human model that has 1,700 million meshes is revolutionary in huge-scale analyses.

II. Parallel Algorithms for Solving Large Linear System

A. Domain Decomposition Method

The domain decomposition method (DDM) is introduced to high-frequency problems using the **E** method [1]. Let us put the finite element equations [1] in matrix form, as follows:

$$Ku = f, \tag{1}$$

where K denotes the coefficient matrix, u the unknown vector, and f the known right-hand side vector. As shown in the following equation, the domain Ω is decomposed into N pieces so that there is no overlap in the boundary between subdomains, namely

$$\Omega = \bigcup_{i=1}^{N} \Omega^{(i)} . \tag{2}$$

After domain decomposition, (1) is rewritten as follows:

$$\begin{bmatrix} K_{II}^{(1)} & \cdots & 0 & & K_{IB}^{(1)} R_B^{(1)} \\ 00 & \ddots & \vdots & & \vdots \\ \vdots & & \ddots & 0 & \vdots \\ 0 & \cdots & 0 & K_{II}^{(N)} & K_{IB}^{(N)} R_B^{(N)} \\ R_B^{(1)T} K_{IB}^{(1)T} & \cdots & \cdots & R_B^{(N)T} K_{IB}^{(N)T} & \sum_{i=1}^{N} R_B^{(i)T} K_{BB}^{(i)} R_B^{(i)} \end{bmatrix} \begin{bmatrix} u_I^{(1)} \\ \vdots \\ \vdots \\ u_I^{(N)} \\ u_B \end{bmatrix} = \begin{bmatrix} f_I^{(1)} \\ \vdots \\ \vdots \\ f_I^{(N)} \\ f_B \end{bmatrix} \tag{3}$$

where the subscripts I and B correspond to unknowns in the interior of subdomains and on the interface boundary, respectively. $R_B^{(i)}$ maps the global degrees of freedom (DOF) of the interface onto the local DOF of the subdomain interface. Equation (3) leads to linear systems as follows:

$$K_{II}^{(i)} u_I^{(i)} = f_I^{(i)} - K_{IB}^{(i)} R_B^{(i)} u_B , \; i = 1, \ldots, N , \tag{4}$$

$$S \, u_B = g , \tag{5}$$

$$S = \sum_{i=1}^{N} R_B^{(i)T} S^{(i)} R_B^{(i)} , \; S^{(i)} = K_{BB}^{(i)} - K_{IB}^{(i)T} \left(K_{II}^{(i)} \right)^{-1} K_{IB}^{(i)} , \tag{6}$$

$$g = \sum_{i=1}^{N} R_B^{(i)T} g^{(i)} , \; g^{(i)} = f_B^{(i)} - K_{IB}^{(i)T} \left(K_{II}^{(i)} \right)^{-1} f_I^{(i)} . \tag{7}$$

We call (4) the subdomain problems, (5) the interface problem, S the Schur complement matrix. In this paper, (4) are solved by the direct method based on Gaussian elimination, on the other hand, (5) is solved by the iterative methods. Here, the Krylov subspace methods require the coefficient matrix-vector multiplication in each iterative procedure, however, such calculation for (5) can be replaced by likely solving subdomain problems. Therefore in our DDM algorithm, subdomain problems are also solved at each iteration for solving the interface problem.

B. COCR Method

The preconditioned COCR method [2] is shown in Fig. 1 [3]. The COCR method requires more or less the same amount of computational cost and working memory per iteration as the COCG method, and is expected to get smooth convergence behavior compared with the COCG method. In particular, the

COCR method for (5) has more effect than the COCG method in a large-scale analysis.

u_B^0 : an initial guess
$r^0 = g - Su_B^0$
$t^0 = M^{-1}r^0$
$p^{-1} = 0,\ q^{-1} = 0,\ \beta^{-1} = 0$
$s^0 = St^0$
$for\ n = 0,1, \ldots$
$p^n = t^n + \beta^{n-1}p^{n-1}$
$q^n = Sp^n = s^n + \beta^{n-1}q^{n-1}$
$w^n = M^{-1}q^n$
$\alpha^n = (\bar{t}^n, s^n)/(\bar{q}^n, w^n)$
$u_B^{n+1} = u_B^n + \alpha^n p^n$
$t^{n+1} = t^n - \alpha^n w^n$
$if\ (\|r^{n+1}\|/\|r^0\| < \delta)\ break;$
$s^{n+1} = St^{n+1}$
$\beta^n = (\bar{t}^{n+1}, s^{n+1})/(\bar{t}^n, s^n)$

Fig. 1. Algorithm of preconditioned COCR method.

III. Numerical Experiments

A. Analysis conditions

The electromagnetic field in the human body is analyzed via refined meshes of 4mm small-scale model. All computations are performed using Oakleaf-FX10 supercomputer at the University of Tokyo. We analyzed frequencies of 1 (MHz), 8 (MHz), 70 (MHz) and 300 (MHz) using the full-wave electromagnetic field analysis based on the DDM. The electromagnetic field source is a dipole antenna assumed to have a current source of 0.8 (A). In the analyses using NICT numeric human body models[1], it is necessary for the user to provide the electric conductivity and the permittivity of the material ID for each internal organ [1].

B. Performance evaluation

Full-wave electromagnetic analyses are performed using the 2mm model that is refined 4mm model once. The model has 220 million tetrahedral elements. Computations are performed using 75-nodes (150 processes x 8 threads) of the Oakleaf-FX10. The iteration count and calculation time for each frequency comparing COCG method and COCR method are shown in Table I.

The number of iterations required by the DDM is largest for the lowest frequency (1 MHz) in each iterative method. Generally, the matrix equation becomes semi-definite in the full-wave electromagnetic analysis based on the vector wave equation. However, when the frequency becomes lower, the matrix equation becomes non-definite. Both results indicate this thing. Fig. 2 shows convergence histories in case of 1 (MHz) using 2mm model. As a result, the iterative calculation is improved drastically by using COCR method as interface solver in DDM. In the previous research by Sogabe and Chou, COCR method solves the matrix which size is 2,534 x 2,534 [2]. In this result, the number of iteration is reduced from the result by using COCG method. On the other hand, more conspicuous improvement is seen by using COCR method in the interface problem with the anatomical human model.

Then, to confirm performances of the huge-scale analysis, the analyses in the smallest frequency are performed using 1mm model that is refined 4mm twice as the most difficult case of the computation. The model has 1,760 million tetrahedral elements. Computations are performed using 600-nodes (1,200 processes x 8 threads) of the Oakleaf-FX10. The iteration count and calculation time in 1 MHz comparing COCG method and COCR method are shown in Table II. Fig. 3 shows convergence histories. We succeed calculation using huge-scale model that has 1,760 million tetrahedral elements. The iterative calculation is also improved by using COCR method in this case.

TABLE I
NUMERICAL RESULT USING 2MM MODEL

Frequency (MHz)	Iter. method	Num. of Iter.	Time (sec)	Peak (%)
1	COCG	13,798	10,330	4.97
	COCR	477	465	3.86
8	COCG	1,383	1,107	4.65
	COCR	221	278	3.02
70	COCG	793	675	4.38
	COCR	128	197	2.50
300	COCG	747	628	4.43
	COCR	86	160	2.09

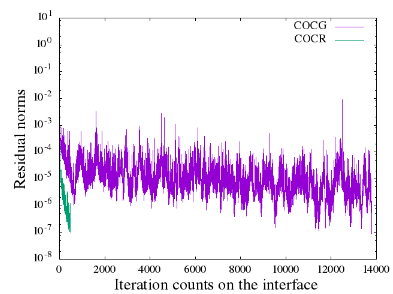

Fig. 2. Convergence histories in case of 1 MHz using 2mm model from once refined 4 mm model.

TABLE II
NUMERICAL RESULT USING 1MM MODEL

Frequency (MHz)	Iter. method	Num. of Iter.	Time (sec)	Peak (%)
1	COCG	7,348	7,071	4.37
	COCR	1,189	1,319	3.82

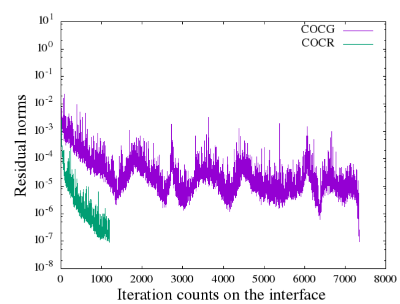

Fig. 3. Convergence histories in case of 1 MHz using 1mm model from twice refined 4mm model.

References

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